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ABSTRACT

The purpose of the two studies reported was to develop and validate clinical assessment instruments designed to identify both the progress of learners through a hierarchy of mathematics content and the mode of presentation through which learners can best handle the content. In Study 1, 73 second graders were given one of two forms of a 15-item instrument assessing knowledge of place value through three-digit numbers. Five modes of symbolic representation were used, ranging from concrete to abstract. When students handled materials or drew pictures, performance was significantly better than when they watched the examiner. No significant differences were found at the concrete-abstract, pictorial-abstract, and abstract levels. In Study 2, 45 sixth graders were given a 48-item instrument assessing knowledge of subtraction of fractions and mixed numbers, using concrete, pictorial, or abstract representations, or all three. The content hierarchy was found to become increasingly difficult from beginning to end of the test. Approximately 92% of the variance was accounted for by itmes using the pictorial mode. Performance was significantly related to ability level. (MNS)

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Assessing Mathematics Understanding: Combining Task-Analytic Conceptual Hierarchies and Modes of Representation

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Paper presented at the Southwest Educational Research Association meeting in Austin, Texas, February 11, 1982

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Many traditional mathematics tests measure student understanding only at the abstract or symbolic level. If students perform poorly on such tests, the usual interpretation is that students do not know the content. The results may state clearly what is known at an abstract level, without judging the students' ability to deal with the content on a more concrete level. In addition, these tests may fail to take into account whether students have mastered prerequisite skills for the concepts included in the test. There may be little to suggest a precise point at which instruction should begin.

In some instances such tests are appropriate. For helping a teacher know how to plan for instruction, however, the tests are helpful only in a general way. What is needed is an assessment device that would indicate both the math content that needed to be taught next and the form of presentation that would make the content most understandable to the student.

The developmental theory of Piaget provides a way to sequence the physical form in which elementary mathematics curricula can be presented. From the concrete embodiment of a quantitative idea through a series of increasingly symbolic levels to complete abstraction, there is a hierarchy of modes of representation that approximate the way in which the cognitive abilities of children develop during their school years. These modes of representation can be translated directly into certain choices of teaching materials and strategies. There have been studies of mathematics performance focusing generally on the use of different modes of

representation as a treatment variable. Results have been mixed both from studies comparing modes and different sequences of modes (Sowder, 1980).

For many years, task analyses of mathematics concepts have been made to specify the sequence in which the concepts are learned (Gagné, 1970; Novillis, 1976). While this approach is certainly commendable and has been widely used to determine the order of the content in mathematics curricula, there have been few attempts to combine task-analytic sequences of mathematics concepts with a developmental sequence of modes of representation. By combining the careful sequencing of mathematics concepts and the hierarchy of modes of representation in mathematics assessment, it is expected that test results could provide not only what content is or is not mastered, but also clues about how that content is most effectively processed by the learner.

This report contains information about two exploratory research projects that investigate the cognitive processes involved in learning mathematics concepts. The purpose of these studies was to develop and validate clinical assessment instruments designed to identify both the progress of learners through a hierarchy of mathematics content and the mode of presentation through which learners can best handle the content.

In addition, the first study investigated whether there were differences in accuracy of performance on mathematics tasks across the modes of representation, between groups of learners given passive or interactive experiences during the assessment. The

second study was designed to see which mode of representation gives the most information about student ability to handle content and to see if teacher perceptions of student ability predicted actual performance.

STUDY 1

Subjects

Ss in this study were 73 second graders selected by a stratified random sampling technique from 15 elementary schools within an urban school system in East Texas. These children represented a cross section of socioeconomic levels, races, and ability levels. Ss were selected by eight elementary curriculum consultants who were trained in selection and examination procedures.

Instrumentation

Place value was selected as the content for the first instrument that was constructed. A 15 item instrument was designed to assess knowledge of place value through three-digit numbers. Place value knowledge of this complexity is a common objective of state-adopted second grade textbook materials in mathematics. Five modes of symbolic representation were used in the place value items, incorporating a hierarchy of increasing abstraction, as illustrated below:

Mode 1 - Concrete. Math tasks require use of manipulative

materials that physically represent numerical quantities. Examples are single sticks and bundles of 10 sticks to represent tens and ones.

Mode 2 - Pictorial. Tasks contain pictures that physically represent the quantity of numbers. An example is squared paper.

Mode 3 - Concrete-abstract. Tasks involve objects that are assigned a value that represents the quantity of some number, but objects do not possess the physical characteristics of the quantities. Examples include token exchanges or money.

Mode 4 - Pictorial-abstract. Pictures of objects used in the mathematics tasks are assigned numerical values, however, objects pictured do not show the physical properties of the quantities they represent. Examples are pictures of a spike abacus or pictures of money.

Mode 5 - Abstract. Symbols are assigned values to represent the quantity of numbers. Numerals are an example.

Two forms of the instrument were made. The first form permitted the examinee only passive contact with the materials. In effect, the subject was an observer, while the examiner manipulated materials or presented pictures to which the subject responded. At the pictorial-abstract level, for example, the learner might be shown a picture of a spike abacus containing

counters on the spikes, and would be asked to or write the number that was shown. The second form, while assessing the same knowledge, allowed subjects to interact with materials. A comparable example for Form 2 would require the subject to draw counters on a picture of a spike abacus to show some particular number.

Form 1 was administered to 40 subjects, and Form 2 was given to 33 students. Each subject was examined individually by one of eight elementary school curriculum consultants. All examiners participated in a one hour training session. At this session, the purpose of the study was explained to the consultants and they received detailed instruction in the administration and scoring of both forms of the instrument. Procedures for selecting subjects by random methods were explained and individuals were given opportunity for additional help if they needed it. Testing took place over a period of four weeks during the spring semester of 1981.

Results

The responses to both forms of the instrument were examined for consistency across items and across examiners. Cronbach's alphas for Forms 1 and 2 were .86 and .90 respectively. Interrater reliability was .88 for both forms.

Subject performance on the two forms of the test was compared using a regression approach to analysis of variance. Performance under the interactive condition was significantly better than

performance under the passive condition for the concrete and pictorial modes of representation with $F(1,71) = 6.23$ and 10.65, $R^2 = .08$ and .13, $p = .015$ and .002, respectively for conrete and pictorial. No significant differences were found between Forms 1 and 2 at the concrete-abstract, pictorial-abstract, and abstract levels.

Concluding remarks

The instrument developed for this project gives reliable information about subjects' knowledge of place value through five modes of representation. In clinical assessment of this content at concrete and pictorial levels, the results indicate that students should work directly with materials. However, at more advanced stages there is no advantage in having individuals use materials. These findings suggest that students needing the concrete/pictorial models are likely to perform better if they can touch as well as see the representations. On the other hand, if students have moved to the concrete-abstract or pictorial-abstract modes, it may be more time efficient to simply use demonstrations.

STUDY 2

Subjects

Sixth-graders (N= 45) were the subjects for this study. These students were selected by teachers on the basis of the researchers' requests for subjects at "high," "average,"

and "low" ability levels. Ss came from three middle schools in one East Texas urban system and from a fourth middle school in a neighboring town. The schools are generally representative of the population in this region. Students from both lower- and middle-class socioeconomic classes and from minority as well as majority races were included in roughly the proportion that they exist in the population.

Instrumentation

The instrument constructed for this study assessed knowledge of subtraction of fractions and mixed numbers. A task analysis of this content showed the following hierarchy:

Fractions and mixed numbers with like denominators.
Examples: $\frac{5}{6} - \frac{1}{6}$, $2\frac{2}{3} - 1\frac{1}{3}$

Fractions with unlike denominators in which one denominator is the lowest common denominator (LCD). Examples: $\frac{5}{10} - \frac{1}{5}$, $\frac{3}{4} - \frac{1}{8}$

Fractions with unlike denominators in which neither denominator is the LCD. Examples: $\frac{3}{4} - \frac{2}{3}$, $\frac{2}{3} - \frac{1}{2}$

Mixed numbers with like denominators in fractions, regrouping a whole number to a fraction required.
Examples: $2\frac{1}{4} - \frac{3}{4}$, $1\frac{2}{8} - \frac{5}{8}$

Mixed numbers with unlike denominators, with or without regrouping. Examples: $3\frac{1}{4} - \frac{1}{2}$, $2\frac{3}{5} - \frac{1}{10}$

Four items were constructed for the first level in each of three modes of representation: concrete, pictorial, and abstract,

yielding 12 items. Then three items were made for each of the remaining levels of the content hierarchy using all three modes of representation. This resulted in an instrument with 48 items.

A graduate research assistant with knowledge of mathematics education and developmental psychology was trained to administer this instrument. She gave the assessments individually to all 45 subjects over a six week period in the spring of 1981.

Results

Responses to the instrument were examined for consistency across items. A Cronbach's alpha for the subtraction test was .93. The results supported the levels of the content hierarchy as becoming increasingly difficult from the beginning to the end of the subtraction test.

A regression analysis was conducted to find which mode of representation provides the best information about student ability. Total scores on the subtraction test were used as the criterion variable and composite scores for each of the modes of representation as the predictor variables. The results showed that approximately 92% of the variance was accounted for by variations in items using the pictorial mode.

Performances of students by teacher perceptions of ability levels were compared using a regression approach to analysis of variance. Performance was significantly related to ability level, $F(1,43) = 12.13$, $R^2 = .22$, $p < .01$

Concluding remarks

The results indicate that the subtraction of fractions and mixed numbers test gives reliable information about students' capabilities in this content area. Items using the pictorial mode of representation offer the best measure by which to judge students' overall ability in subtraction of fractions and mixed numbers.

Student performances did seem to be related to their ability level, according to the results of this study. Of course there is not a way to determine whether students performed as they did because of their group or whether they were in their respective ability groups because of their performance. It is simply interesting to note that students did perform according to the expectations that were held for them.

References

Gagne, R. M. The conditions of learning (2nd ed.). New York: Holt, Rinehart and Winston, 1970.

Novillis, C. F. An analysis of the fraction concept into a hierarchy of selected subconcepts and the testing of the hierarchical dependencies. Journal for Research in Mathematics Education, 1976, 7, 131-44.

Sowder, L. K. Concept and principle learning. In R. J. Shumway (Ed.), Research in mathematics. Reston, Virginia: National Council of Teachers of Mathematics, 1980.